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MECHANISMS AND MODELING OF WIND-INDUCED LOW-FREQUENCY AMBIENT S--ETC(U)  
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# Mechanisms and Modeling of Wind-Induced Low- Frequency Ambient Sea Noise

Nai-Chyuan Yen

Anthony J. Perrone

Special Projects Department

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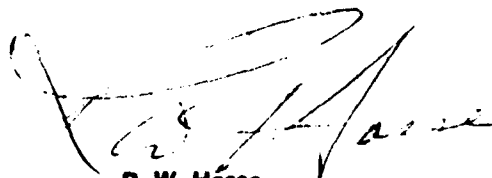
## PREFACE

This report was prepared under NUSC Project No. A65070, "Wind Induced Noise Levels Below 10 Hz," Principal Investigator, N. Yen (Code 312), Program Manager, H. Bezdek (ONR 480), and under NUSC Project No. 65062, "Wind Induced Noise Below 12 Hz," Principal Investigator, A. Perrone (Code 312), Sponsoring Activity, LRAPP, Program Manager, R. Gaul.

The authors acknowledge several helpful discussions with R. Mellen (Code 313) during the progress of the investigation.

The Technical Reviewer was P. Cable, Code 313.

**REVIEWED AND APPROVED:** 13 February 1979

A handwritten signature in dark ink, appearing to read 'R. W. Hasse', is written over a horizontal line.

**R. W. Hasse**  
**Head, Special Projects Department**

The authors of this report are located at the New London  
Laboratory, Naval Underwater Systems Center,  
New London, Connecticut 06320.

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER N 5 -TR-5833	2. GOVT ACCESSION NO. AD-A103 832	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) MECHANISMS AND MODELING OF WIND-INDUCED LOW-FREQUENCY AMBIENT SEA NOISE		5. TYPE OF REPORT & PERIOD COVERED Final Report
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Nai-Chyuan Yen Anthony J. Perrone		8. CONTRACT OR GRANT NUMBER(s) A-650-70 65062
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Underwater Systems Center New London Laboratory New London, CT 06320		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Office of Naval Research Arlington, VA 22217		12. REPORT DATE 13 February 1979
		13. NUMBER OF PAGES 36
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report)  Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Sea Noise, Ambient Wind-Induced Ambient Sea Noise Low-Frequency Ambient Sea Noise Wind Turbulence, Ocean Surface Wave, Ocean Wave Turbulence, Ocean Ocean Wave Turbulence and Noise		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) Wind turbulence, surface waves, and wave turbulence have been examined as the noise-generating mechanisms for ambient noise in the 1 to 10 Hz range in the deep ocean. The theoretical derivation, based on stationary ocean surface disturbances, formulates a relationship between wind speed and ambient noise level. The predicted level attributable to these three mechanisms agrees in order of magnitude with the measured data obtained in the Bermuda and Grand Banks areas. Analyses of the noise-level spectrum and (over)		

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→ wind speed variations have also been discussed to determine the dominant parameters affecting the wind-induced noise. A form of semi-empirical wind noise model is suggested as a reference for data comparison and a guideline for ambient noise measurement.

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## MECHANISMS AND MODELING OF WIND-INDUCED LOW-FREQUENCY AMBIENT SEA NOISE

### INTRODUCTION

During the investigation of the characteristics of ambient sea noise, it has been observed that certain portions of the noise spectrum level are related to local wind conditions. Knudsen curves were established in the later 1940's by grouping the noise spectrum level according to sea state.<sup>1</sup> A generalized wind noise spectrum was compiled in Wenz's survey paper and was supported by the results of many measurements in the frequency range above 200 Hz.<sup>2</sup> Below 200 Hz it was believed that ambient sea noise is dominated by nonwind dependent sources, such as shipping, biological, and industrial noises. However, recent analysis of ambient noise data below 10 Hz indicates that the noise level in this part of the spectrum also has a considerable correlation with windspeed.<sup>3</sup> This particular relationship has not been fully explored.

There are many hypotheses being proposed to explain the causes of ambient sea noise attributed to surface wind conditions. The noise-generating mechanisms derived from theory can generally be summed up in three categories. One treats wind turbulence above the ocean surface.<sup>4,5,6</sup> The second category deals with surface-wave interaction.<sup>4,7-11</sup> and the third treats surface wave and turbulence interaction.<sup>12</sup> Because the theoretical formulations of such problems are based on assumptions that have little supporting experimental data (particularly in an ocean environment), these theoretical results are not adequate to interpret the measurement noise data collected at various conditions in the actual ocean environment. Definite conclusions cannot be reached because there is no direct correspondence between measurement and theory. Noise data below 10 Hz are especially lacking. Furthermore, attempts to use one type of mechanism to explain the entire noise spectrum appear to be an oversimplification.

This report reviews some of the basic sound-generating mechanisms attributed to ocean surface disturbances caused by the wind. The report derives the ambient noise level attributed to such causes and compares the theoretical results with measurements obtained in known ocean environments. The study mainly concerns itself with the wind-induced noise spectrum below 10 Hz because a reliable ambient noise data set with supporting environment information has been reported recently and is available.<sup>13</sup> When considering only the low frequency range, some of the formulation of noise-generating mechanisms can be simplified with physical argument.



The analysis discussed in this report extends from previous work, but the statistical nature of ambient sea noise is emphasized instead of deriving a deterministic model.<sup>4-11</sup> It also takes into consideration the ocean bottom, water depth, and sensor depth so that the predicted theoretical noise spectrum level will closely correspond to the measured data obtained in the actual ocean environment. Physical arguments will be used to simplify some mathematical operations and to understand the connection between the noise-generating mechanism and the noise spectrum level. This also provides an understanding of the conclusions from this study and suggests how far the result will remain valid if conditions change from those assumed in the derivation.

A wind-induced noise model will be suggested as a result of this study to highlight the important parameters of such a noise source and to provide a basis for the computation of noise fields in computer modeling. It also can serve as a reference for data comparison and as a guide for future planning in ambient noise measurements.

#### FUNDAMENTALS AND FORMULATION

The mathematical description of the acoustic wave is based on the physics of fluid mechanics.<sup>14</sup> To identify the important parameters linking the ocean surface disturbance caused by wind with the ambient sea noise level detected by a sensor in the water column, the fundamental laws of fluid mechanics are reviewed here. Formulation of the wind-induced ambient sea noise is based on this derivation.

According to the conservation laws of mass and momentum of fluid mechanics, the relationships among the density of fluid  $\rho'$ , its particle velocity  $u'$ , pressure  $p'$ , and the viscosity  $\nu$  (shear),  $\nu'$  (dilatation) can be described by an equation of continuity,

$$\frac{\partial \rho'}{\partial t} + u'_i \frac{\partial \rho'}{\partial x_i} + \rho' \frac{\partial u'_i}{\partial x_i} = 0, \quad (1)$$

and a force equation,

$$\rho' \frac{\partial u'_i}{\partial t} + \rho' u'_j \frac{\partial u'_i}{\partial x_j} = - \frac{\partial p'}{\partial x_i} + \frac{\partial}{\partial x_j} [\nu' d_{kk} \delta_{ij} + 2\nu d'_{ij}], \quad (2)$$

where  $i, j, k$  are coordinate index for coordinates 1, 2, 3, i.e.,

$$d_{ij} = \frac{1}{2} \left[ \frac{\partial u'_i}{\partial x_j} + \frac{\partial u'_j}{\partial x_i} \right],$$

and repeated index denotes summation.

Consider that those variables can be partitioned into two parts, i.e.,

$$u'_i = U_i + u_i,$$

$$\rho' = \rho_0 + \rho,$$

and

$$p' = P + p, \quad (3)$$

where  $U_i$ ,  $\rho_0$ ,  $P$  can be regarded as associated with the flow mode (varied slowly with respect to time and space) and  $u_i$ ,  $\rho$ ,  $p$  the acoustic mode (the fluctuation part of fluid disturbance).<sup>1</sup> In a spatial scale where  $\rho_0$  can be considered to be a constant with respect to time, the flow mode is not compressible. Equation (1) can then be reduced to

$$\frac{\partial U_i}{\partial x_i} = 0 \quad (4a)$$

and

$$\frac{\partial \rho}{\partial t} + \rho_0 \frac{\partial u_i}{\partial x_i} + \frac{\partial \rho}{\partial x_i} U_i + \frac{\partial \rho}{\partial x_i} u_i + \rho \frac{\partial u_i}{\partial x_i} = 0. \quad (4b)$$

From equation (2) we derive

$$\rho_0 \frac{\partial U_i}{\partial t} + \rho_0 U_j \frac{\partial U_i}{\partial x_j} = - \frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j} \left[ \nu' D_{kk} \delta_{ij} + 2\nu D_{ij} \right], \quad (5a)$$

and

$$\begin{aligned} \rho \left( \frac{\partial U_i}{\partial t} \right) + \rho_0 \left( \frac{\partial u_i}{\partial t} \right) + \rho \left( \frac{\partial u_i}{\partial t} \right) + \rho_0 U_j \frac{\partial u_i}{\partial x_j} + \rho_0 u_j \frac{\partial u_i}{\partial x_j} \\ + \rho U_j \frac{\partial u_i}{\partial x_j} + \rho u_j \frac{\partial u_i}{\partial x_j} + \rho_0 u_j \frac{\partial U_i}{\partial x_j} \\ + \rho U_j \frac{\partial U_i}{\partial x_j} + \rho u_j \frac{\partial U_i}{\partial x_j} \end{aligned}$$

$$= - \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left[ v' d_{kk} \delta_{ij} + 2v d_{ij} \right], \quad (5b)$$

where

$$D_{ij} = \frac{1}{2} \left[ \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right],$$

and

$$d_{ij} = \frac{1}{2} \left[ \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right].$$

Equations (4a) and (5a) describe the flow mode and equations (4b) and (5b) show the coupling of flow mode with acoustic mode. Because of equation (4a), flow mode is assumed irrotational and velocity potential,  $\phi$ , can be defined as

$$U_i = - \frac{\partial \phi}{\partial x_i}. \quad (6)$$

The equation of continuity for flow, (4a), becomes Laplace equation

$$\left( \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2} \right) \phi = 0. \quad (7)$$

By neglecting the effect of viscosity in the flow and recalling the irrotationality assumption, we see that equation (5a) can be reduced to

$$\frac{P}{\rho_0} = \frac{\partial \phi}{\partial t} + \frac{1}{2} U^2 + F(t), \quad (8a)$$

which is one form of Bernoulli's equation. For acoustic mode, equations (4b) and (5b) can be combined into a form as

$$\frac{\partial^2 p}{\partial x_i^2} - \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = -\rho_0 \frac{\partial^2 u_i u_j}{\partial x_i \partial x_j} - \rho_0 \frac{\partial^2}{\partial x_i \partial x_j} (u_i u_j) + \frac{\partial^2}{\partial x_i \partial x_j} \left[ v' d_{kk} \delta_{ij} + 2v d_{ij} \right], \quad (8b)$$

where the compressibility of fluid,  $p^2/\rho$ , is assumed to be a constant,  $c^2$ . Later in this report, under an analysis of wind turbulence, equation (8b) can be recognized as an inhomogeneous wave equation with right-hand side terms as noise sources or sinks. For example, the term

$$\rho_0 \frac{\partial^2}{\partial x_i \partial x_j} (u_i u_j)$$

is called the Reynolds Stress and is responsible for noise generated by turbulence.<sup>15</sup> It depends on the order of magnitude for variables in equation (8b) (initial condition) and the environment of this equation (boundary conditions). Further simplification of this equation can be made according to the situation where it applies. The above derivation not only defines the notations used in the analysis of this paper but also emphasizes the connection between flow and the acoustic wave.

To analyze the wind-induced low frequency ambient sea noise based on the fundamentals of fluid mechanics, a simplified scheme of ocean environment (as shown in figure 1) will be used. As this study is mainly concerned with the low-frequency acoustic wave, the physical dimensions will be scaled with reference to wavelength. For example, at 10 Hz the wavelength of the acoustic wave will be 150 m whereas the corresponding wavelength of the surface wave (the gravity wave, the capillary wave will not be appreciable for  $f < 13.5$  Hz) is about 0.017 m ( $\Lambda = 2\pi g/\Omega^2$ ). The boundary conditions that apply to flow and to the acoustic wave can be approximated in a different way. To simplify the mathematical analysis, the whole scheme shown in figure 1 is divided into three regions. In region I, the direction of wind is assumed to be parallel with the  $x_1, x_2$  plane. Since  $\delta$  is larger than the boundary layer thickness ( $\delta \gg 5.5 \nu/U$ ), the wind condition above  $\delta$  can be considered<sup>16</sup> as a laminar flow. Region II contains air-water interface. The depth of disturbance in this region,  $d$ , is much smaller than the acoustic wavelength, but is about the same order as the wavelength of the surface wave. Region III is the water column with a semi-infinite bottom layer. In figure 1,  $\rho$  and  $c^2$  designate the density and compressibility, respectively, in each medium with subscript "a" for air, "b" for bottom layer, and no subscript for water. Channel depth,  $D$ , and sensor location,  $s$ , are also displayed in figure 1 so the computed noise field can be closely compared to the measurement data.

In this analysis, the behavior of fluid in region I can be simply described by equation (4a), and the wind speed condition provides a boundary condition. The noise is mainly generated at the air-water interface in region II. The induced noise occurring at region II will then be propagated into region III. Because  $d$  is only a fraction of acoustic wavelength in water, the pressure disturbance on the air-water interface can be regarded as the boundary condition at  $x_3 = d$ . Since  $d$  is also approximately the order of the wavelength for the surface wave, the hydrostatic pressure caused by the undulation of the ocean surface will be negligible at this depth. The derivation that relates disturbances on the ocean surface to ambient sea noise is based on this simplification.

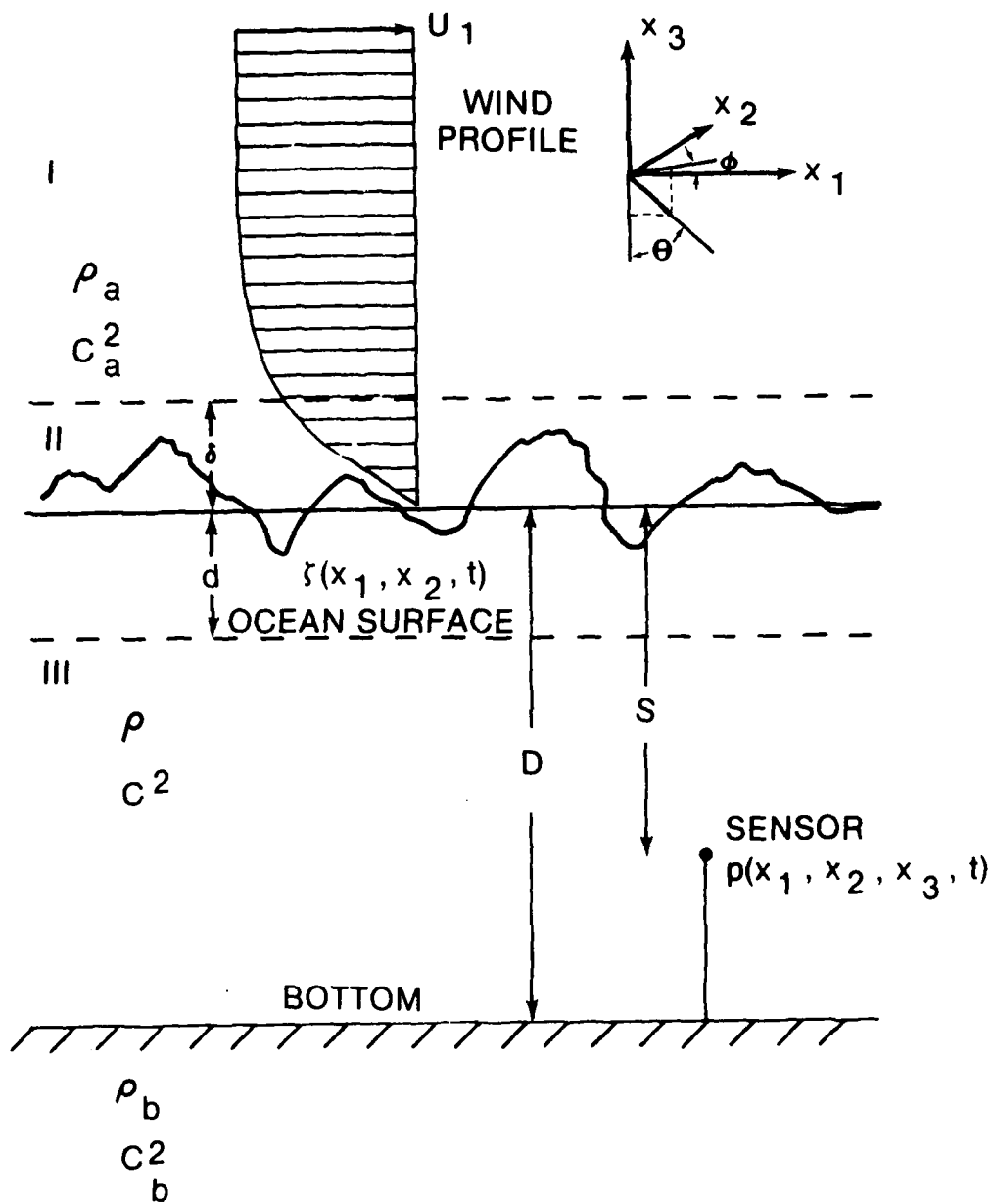


Figure 1. Analysis of Wind-Induced Noise

## NOISE-GENERATING MECHANISMS

Wind Turbulence

The existence of an air-water interface will cause the laminar flow of wind to change to a turbulence flow. The physical processing of this phenomenon is rather complicated because many parameters, such as undulated boundary, temperature difference between air and water, viscosity, and evaporation rate of water should all be taken into consideration.<sup>17</sup> However, the problem we are concerned with here is that of the disturbance occurring in the 1 to 10 Hz frequency range where the vorticity size is small. Some understanding about isotropic turbulence can be utilized to simplify the mathematical treatment of the wind turbulence problem above the sea surface.<sup>18</sup>

Because the purpose of analyzing wind turbulence is to determine the pressure distribution at the interface, equation (8b) can be used to derive the relationship of pressure and fluctuation of velocity. The acoustic wavelength is very large compared with the dimension of the turbulence boundary layer, so the retardation of wave will be ignored and the interface will be considered as  $x_3 = 0$ . Therefore, equation (8b) can be rewritten as

$$\frac{\partial^2 p_a}{\partial x_i^2} - \rho_{ao} \frac{\partial^2 u_{ai} u_{aj}}{\partial x_i \partial x_j} - \rho_{ao} \frac{\partial^2}{\partial x_i \partial x_j} (u_{ai} u_{aj}) = -\rho_{ao} s(\bar{x}, t) \quad (9)$$

By assuming  $\partial p / \partial x_3 \approx 0$  around the boundary  $x_3 = 0$ , we see that the pressure distribution can be expressed as

$$p(x_1, x_2, 0, t) = \frac{\rho_{ao}}{2\pi} \int_0^\delta \int_{-\infty}^\infty \int_{-\infty}^\infty \frac{S(\bar{z}, t)}{|\bar{x} - \bar{z}|} d\bar{z} \quad (10)$$

where  $\delta$  is the thickness of the turbulent flow. Because  $s(\bar{z}, t)$  is not known exactly, only the statistical property of  $p$  can be derived. This problem has been treated based on the theory of isotropic turbulence.<sup>19,20</sup> The following are the major steps in the mathematical operation and are adapted with present notations to implement physical interpretation in the derivation of the wind-induced noise level computation.

First assume that the turbulence field,  $U$ 's, is isotropic (Kolmogoroff's hypothesis) and, then, the time<sup>a</sup> average of the spatial- and-time correlation for  $S$  will depend only on the relative separation<sup>18</sup>

$$\overline{S(\bar{Z}, t) S'(\bar{Z}', t')} = F[|\bar{Z} - \bar{Z}' - U_1 \zeta|, |Z_3 - Z'_3|, \frac{1}{2} |Z_3 + Z'_3|], \quad (11)$$

$$= F[|\bar{Z} - \bar{Z}' - U_1 \zeta, \xi, z|],$$

$$= F[|\bar{\gamma} - \bar{\gamma}' + \bar{Z}_3 - \bar{Z}'_3 - U_1 \zeta|, \xi, z],$$

and

$$= F[|\bar{\gamma} - \bar{\gamma}' - U_1 \zeta|, \xi, z],$$

where  $\bar{\gamma}$ 's is the horizontal separation between the boundary point and the source point in the turbulence boundary layer,  $U$  is the mean velocity of the flow at  $1/2 (Z_3 + Z'_3)$ , and  $\tau$  is the time delay. Then equation (10) can be extended as follows:

$$R_p(\bar{x}, \bar{x}', t, t') = \overline{p(x_1, x_2, 0, t) p(x'_1, x'_2, 0, t')} \quad (12)$$

$$= \frac{\rho_{ao}^2}{4\pi^2} \iint \frac{\overline{S(\bar{Z}, t) S(\bar{Z}', t')}}{|\bar{x} - \bar{Z}| |\bar{x}' - \bar{Z}'|} d\bar{Z} d\bar{Z}'$$

$$\approx \frac{\rho_{ao}^2}{4\pi^2} \int_0^\delta dZ_3 dZ'_3 \int_{-\infty}^\infty \int_{-\infty}^\infty \frac{d\bar{\gamma} d\bar{\gamma}' F[|\bar{\gamma} - \bar{\gamma}' - U_1 \zeta|, \xi, z]}{\sqrt{|\bar{x} - \bar{\gamma}|^2 + Z_3^2} \sqrt{|\bar{x}' - \bar{\gamma}'|^2 + Z_3'^2}}$$

$$= \frac{\rho_{ao}^2}{4\pi^2} \int_0^\delta dZ_3 dZ'_3 \int_{-\infty}^\infty \int_{-\infty}^\infty \frac{d\bar{\gamma} d\bar{\gamma}' F[|\bar{\gamma} - \bar{\gamma}' + \bar{x} - \bar{x}' - U_1 \zeta, \xi, z|]}{\sqrt{|\bar{\gamma}|^2 + Z_3^2} \sqrt{|\bar{\gamma}'|^2 + Z_3'^2}}$$

Applying the relationship of Hankel transforms yields

$$\int_0^\infty d\bar{\gamma} d\bar{\gamma}' G_1(\bar{\gamma}) G_2(\bar{\gamma}') F(\bar{\gamma} - \bar{\gamma}' - x) = 4\pi^2 \int_0^\infty K dK g_1(K) g_2(K) f(K) J_0(K|\bar{x}|), \quad (13)$$

where  $G$  and  $g$  are Hankel transforms pairs

and

$$g(K) = \int_0^{\infty} \gamma d\gamma J_0(K\gamma) G(\gamma)$$

$$G(\gamma) = \int_0^{\infty} K dK J_0(K\gamma) g(k)$$

Equation (12) has the form

$$R_p(\xi, \eta, \tau) = 2p_{ao}^2 \int_0^{\delta} dz \int_0^{2z} d\zeta \int_0^{\infty} \frac{dK}{K} e^{-2Kz} f(K, \zeta, z) \cdot J_0[K \sqrt{(\xi - U_1 \tau)^2 + \eta^2}] , \quad (14)$$

where, based on the assumption that the disturbance is stationary in the  $x_1$  and  $x_2$  direction, relations of

$$\xi = |x_1 - x_1'| ,$$

$$\eta = |x_2 - x_2'| , \quad (15)$$

$$f(K) = \int_0^{\infty} \gamma d\gamma J_0(K\gamma) F(\gamma) ,$$

and

$$\int_0^{\infty} \frac{\gamma d\gamma J_0(K\gamma)}{\gamma^2 + a^2} = \frac{e^{-Ka}}{K}$$

have been used.

The power spectrum of  $p$  can be derived from the Fourier transform of  $R_p$ :

$$\begin{aligned} \phi_{pp}(\xi, \eta, \omega) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} R_p(\xi, \eta, \tau) e^{-i\omega\tau} d\tau \\ &\approx \frac{2p_{ao}^2}{\pi} \int_0^{\infty} \frac{dz}{U_1} e^{\frac{-i\omega\xi}{U_1}} \int_0^{2z} d\zeta \int_{\omega/U_1}^{\infty} \frac{dK e^{-2Kz} f(K, \zeta, z)}{K \sqrt{K^2 - (\omega/U_1)^2}} \quad (16) \end{aligned}$$



$$\cdot \cos (\eta K^2 - (\omega/U_1)^2)$$

as

$$\int_0^{\infty} J_0 [a(x^2 + b^2)^{1/2}] \cos(xy) dx = (a^2 - y^2)^{-1/2} \cos[b(a^2 - y^2)^{1/2}] \text{ for } 0 < y < a$$

Changing the variable

$$x^2 = \left( \frac{KU_1}{\omega} \right)^2 - 1$$

and replacing  $f(K, \xi, z)$  by the average value in the turbulence boundary layer  $f_{avg}$ , equation (16) can be further simplified to

$$\begin{aligned} \phi_{pp}(\xi, \eta, \omega) &= \frac{4\rho_{ao}^2 f_{avg}}{\pi\omega} \int_0^{\delta} z dz e^{-i\omega\xi/U_1} \\ &\cdot \int_0^{\infty} \frac{e^{-\frac{2\omega z}{U_1} \sqrt{1+X^2}}}{1+X^2} \cos \frac{\omega\eta X}{U_1} dX, \end{aligned} \quad (17)$$

where  $\delta$  is the displacement of the turbulent boundary layer. To complete the integration, the mean velocity profile of  $U_1$  is needed. White used the relationship

$$\frac{U_1}{U_{\infty}} = \frac{z}{\delta}^{1/7} \quad (18)$$

to obtain the numerical result of equation (17). It agrees well with the wind tunnel experiment.<sup>21,22,23</sup> For the problem we are concerned with here, it is desirable to have a close-form results derived from equation (17) so that the noise field in the water column can be computed. Since it is not easy to reduce equation (17) to such a form, an alternative approach is to formulate a semi-empirical expression based on its numerical result and the measured value from the wind-tunnel experiment.<sup>20</sup> The final result is

$$\phi_{pp}(\xi, \eta, \omega) = \rho_{ao}^2 \delta^3 U_{\infty}^3 \frac{1.5 \cdot 10^{-6}}{[1 + 10^{-2} \frac{\omega\delta}{U_{\infty}}]^3}$$

(19)

$$e^{-i(\omega\xi/U_c) - 0.12\omega\xi/U_c - 0.55\omega\eta/U_\infty},$$

where  $U_c$  is the convection velocity. Experimental data indicate<sup>20</sup> that  $U_c$  and free flow velocity  $U_\infty$  can be related by

$$\frac{U_c}{U_\infty} = e^{-0.024 \omega\delta/U_\infty} \quad (20)$$

Fourier transform of equation (19) gives the wave number and frequency spectrum:

$$S_{pp}(\omega, \bar{k}) = \frac{1.5 \cdot 10^{-6} \rho_a^2 U_\infty^3 \delta}{[1 + 10^{-2} \frac{\omega\delta}{U_\infty}]} \quad (21)$$

$$\cdot \frac{\pi}{2} \frac{\frac{0.12\omega}{U_c} \frac{0.55\omega}{U_\infty}}{[\frac{0.12\omega}{U_c}^2 + K_1 + \frac{\omega^2}{U_c^2}] [\frac{0.55\omega}{U_\infty}^2 + K_2]}$$

To use the result of equation (21) in an ocean environment, knowledge of wind profiles above the air-water interface is needed to determine the thickness of the turbulent boundary layer. In general, for an aerodynamically smooth flow, the profile is found by experiment to have a logarithm form above the turbulence boundary layer (the mean velocity profile in the turbulence layer was assumed to follow equation (18) in the derivation for numerical computation).<sup>6,24</sup> However, for a rough surface (such as the ocean surface under windy conditions), the thickness of the turbulence boundary layer probably has to be related to wave height as is observed in a recent study.<sup>25</sup> A good reference, therefore, would be one-third wave height. According to the field data collected during fully developed seas, the turbulence layer is assumed to be

$$\delta = H_{1/3} = 4.426 \times 10^{-3} U^{2.5}, \quad (22)$$

where  $\delta$  is units of feet and  $U_\infty$  is units of knots.<sup>26,27</sup>

Whether or not pressure fluctuations with spectrum of equation (21) cause noise generation in the fluid below depends on the wave number vector  $k$ . Only those disturbances on the surface where  $k^2 \leq (\omega/c)^2$  will generate an unattenuated sound wave into the water column of region III. Computation of the noise field attributed to such a noise generating mechanism will be carried out in a later section.

### Surface Wave

The theory of the mechanisms for wind-induced ocean surface waves has been developed extensively by Phillips and Miles.<sup>28,29</sup> The power spectrum of such waves modeled by Pierson and Moskowitz<sup>30</sup> according to the observed field data has the form

$$I(\omega) = \alpha g^2 \omega^{-5} e^{-\beta(\omega_0/\omega)^4}, \quad (23)$$

where

$$\alpha = 8.10 \cdot 10^{-3},$$

$$\beta = 0.74,$$

$$\omega_0 = g/U, \text{ and}$$

$U$  = wind speed reported by weather ship.

Some of the basic properties of the surface wave can be explained by the linear theory as derived by equations (6) and (8a) in the previous section. If only the gravity force is involved (gravity wave), equation (8a) becomes

$$\frac{\partial \phi}{\partial t} - \frac{1}{2} U^2 - g x_3 = 0, \quad (24)$$

and, taking  $\partial/\partial t$  from it,

$$\frac{\partial^2 \phi}{\partial t^2} - \frac{\partial}{\partial t} (\nabla \phi)^2 + g \frac{\partial \phi}{\partial x_3} = 0. \quad (25)$$

For small disturbances, the effect of quadratic term  $\partial/\partial t (\nabla \phi)^2$  can be neglected and

$$\frac{\partial^2 \phi}{\partial t^2} + g \frac{\partial \phi}{\partial x_3} = 0. \quad (26)$$

By means of equations (7) and (26), the dispersive relation of the gravity wave is obtained, i.e.,

$$gk_3 - \omega^2 = 0, \quad (27)$$

which has a velocity potential of

$$\phi(x_1, x_3, t) = \int b(\omega, \bar{k}) e^{ik_3 x_3 + i(\omega t - \bar{k}_r \cdot \bar{r})} d\omega d\bar{k}, \quad (28)$$

with

$$|\bar{r}| = x_1^2 + x_2^2 \quad \text{and} \quad |k_1^2 + k_2^2| = k_3^2 = |k_r^2|;$$

then  $b$  can be related to  $I(\omega)$  by matching the boundary condition at  $x_3 = \zeta$  (ocean surface) and

$$\left| \frac{1}{\omega} \int k_3 b(\omega, \bar{k}) d\bar{k} \right|^2 = I(\omega) \quad (29)$$

Equation (28) also indicates that pressure due to flow mode (first-order disturbance),  $\rho_0 \partial \phi / \partial t$ , will be attenuated in the water column for  $x_3 < 0$ . Hence, noise detected by a sensor located at  $x_3 \ll -1/k_3$  is not caused by such disturbances.

The second-order effect attributed to ocean surface disturbances can be derived from the acoustic mode with equation (9)\* which can be rewritten as

$$\begin{aligned} \frac{\partial^2 p}{\partial x_i^2} - \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} &= -\rho_0 \frac{\partial^2}{\partial x_i \partial x_j} (U_i U_j) + \dots \\ &= -\rho_0 S(\bar{x}, t), \end{aligned} \quad (30)$$

where  $S(\bar{x}, t)$  is the source term caused by surface disturbance and can be derived from the results of equation (28). Because the general solution of equation (30) has the form

$$p \rightarrow e^{i\omega t - i\bar{k}_r \cdot \bar{r} - ik_3 x_3}, \quad (31)$$

---

\*The quadratic term in flow mode, second order term, is put back in the acoustic mode.

with

$$\frac{\omega^2}{c^2} = |k_r|^2 + k_3^2,$$

or

$$k_3^2 = \frac{\omega^2}{c^2} - |k_r|^2,$$

to have propagated a pressure wave in the  $x_3$  direction, only those with  $|k_r|^2 \leq \omega^2/c^2$  from source  $S(x,t)$  excited such a propagation mode.

The wave number spectrum of the ocean surface wave is not totally understood yet. Experimental data and proposed theoretical explanations have not reached satisfactory conclusions.<sup>31,32,33</sup> The available empirical model is given by an angular distribution, such as

$$h(\theta) = 2^{2s-1} \frac{r^2(s+1)}{r(2s+1)} \left| \cos \frac{1}{2} (\phi - \phi_0) \right|^{2s}, \quad (32)$$

where  $s$  is about 3 for a frequency above 0.3 Hz and  $\phi_0$  is the direction of the wind.<sup>34</sup> When expressing equation (32) in terms of wave number,  $H(k)$  becomes

$$H(k) = 0.2 \left( \frac{|k| + \bar{k} \cdot \bar{n}_1}{|k|} \right)^3, \quad (33)$$

where

$$|k| = \omega^2/g$$

and  $\bar{n}_1$  is a unit vector along the  $x_1$  axis, or the direction of the wind. Combining equations (33) and (23), we get the approximated surface wave spectrum of the wave number and the frequency is modeled as

$$\begin{aligned} E(\omega, \bar{k}) &= I(\omega) H(K) \delta(|K| - |K_0|)/|K| \\ &= \alpha' g^2 \omega^{-5} e^{-\beta(\omega/\omega_0)^4} \left( \frac{|k| + \bar{k} \cdot \bar{n}_1}{k} \right)^3 \delta(|k| - |k'|)/|k|, \end{aligned} \quad (34)$$

where

$$\alpha' = 1.62 \cdot 10^{-3}, \quad K' = \omega^2/g,$$

and other notations are the same as defined previously.

Because this derivation is concerned with the pressure fluctuation in region II with a length scale small in comparison with acoustic wavelength, the equation is simplified to

$$\begin{aligned} \frac{\partial^2 p}{\partial x_i^2} &\approx -\rho_0 \frac{\partial^2}{\partial x_i \partial x_j} (U_i U_j) \\ &= -\rho_0 \frac{\partial^2}{\partial x_i \partial x_j} \left[ \frac{\partial \phi}{\partial x_i} \frac{\partial \phi}{\partial x_j} \right] \end{aligned} \quad (35)$$

Using Fourier-Stieltjes integral representation yields

$$p(\bar{x}, t) = \int dP(k, \omega) e^{i(\omega t - \bar{k} \cdot \bar{r}) + ik_3 x_3} d\bar{k}_r d\omega \quad (36)$$

and

$$\phi(\bar{x}, t) = \int d\phi(K, \omega) e^{i(\omega t - \bar{K} \cdot \bar{r}) + K_3 x_3} d\bar{K} d\omega,$$

where  $\bar{r}$ ,  $\bar{k}$ ,  $\bar{K}$  are considered to be two-dimensional variables only; i.e.,  $\bar{r} = (x_1, x_2)$ , etc. Equation (35) then reduces to

$$\begin{aligned} &\int |k|^2 dP(\omega, \bar{k}) e^{i(\omega t - \bar{k} \cdot \bar{r})} d\bar{k}_r d\omega \\ &= -\rho_0 \int [K_i'^2 K_j'^2 K_j''^2 + K_i'^2 K_j''^2 + K_i' K_j' K_i'' K_j'' + K_i' K_i'' K_j''^2] \\ &\quad \cdot d\phi'(\bar{K}', \omega') d\phi''(\bar{K}'', \omega'') e^{i[(\omega' + \omega'')t - (\bar{K}' + \bar{K}'') \cdot \bar{r}]} \\ &\quad \cdot d\bar{K}' d\bar{K}'' d\omega' d\omega'', \end{aligned} \quad (37)$$

by evaluating the equation at  $x_3 = 0$ . Let

$$\begin{aligned} \omega' + \omega'' &= \omega, \\ \bar{K}' + \bar{K}'' &= \bar{k}_r, \end{aligned} \quad (38)$$

$$\begin{aligned}
|k|^2 dP(\omega, \bar{k}) &= -\rho_0 \int [K'_i K'_j (k_j - K'_j) + K'^2_i (k_j - K'_j)^2 \\
&\quad + K'_i K'_j (k_i - K'_i) (k_j - K'_j) + K'_i (k_i - K'_i) (k_j - K'_j)^2] \\
&\quad \cdot d\phi'(\bar{K}', \omega) d\phi''(\bar{K} - \bar{K}', \omega - \omega') d\bar{K}' d\omega' \\
&= \rho_0 \int [-k_1^2 K_1'^2 + 2k_1 k_2 K_1' K_2' + 2i k_1 k_3 K_1' K_3' \\
&\quad - k_2^2 K_2'^2 + 2i k_2 k_3 K_2' K_3' + k_3^2 K_3'^2] d\phi'(\bar{K}', \omega) \\
&\quad \cdot d\phi''(\bar{K} - \bar{K}', \omega - \omega') d\bar{K}' d\omega'.
\end{aligned} \quad (39)$$

Recall that

$$|k|^2 = \frac{\omega^2}{c^2} \text{ and } K_1'^2 + K_2'^2 = \left| \frac{\omega}{g} \right|^2 = K_3'^2 \quad (40)$$

and

$$|k_r^2| < |k^2|, \quad |k_r^2| \ll |K_1'^2 + K_2'^2|, \quad |k_r|^2 \ll K_3'^2$$

and consider only those waves that propagate downwards:

$$|k_3| > |k_1| \text{ or } |k_2|.$$

Then equation (39) can be further reduced to

$$dP(a, \bar{k}) \approx \rho_0 \frac{k_3^2}{|k|^2} \int |K_3'|^3 d\phi'(K', \omega') d\phi''(\bar{K} - \bar{K}', \omega - \omega') d\bar{K}' d\omega'. \quad (41)$$

Because of the condition of equation (40), it may be assumed that

$$\begin{aligned}
K' &\approx -K'', \\
\omega' &\approx \omega'',
\end{aligned} \quad (42)$$

and

$$\omega = 2\omega' = 2\omega'',$$

with

$$d\bar{K}' = |K'| d|K'| d\phi'.$$

Equation (41) can now be rewritten as

$$\begin{aligned}
 dP(\omega, \bar{k}) &\approx \rho_0 \cos^2 \theta \int K_3'^2 d\phi' (\bar{k}', \omega') d\phi'' (-\bar{k}', \omega') \delta(\omega - 2\omega') d\bar{k}' d\omega' \\
 &= \rho_0 \cos^2 \theta \int K_3'^2 d\phi' (\bar{k}', \frac{1}{2}\omega) d\phi'' (-\bar{k}', \frac{1}{2}\omega) d\bar{k}' \\
 &= \rho_0 \cos^2 \theta \left(\frac{\omega}{2}\right)^2 I\left(\frac{\omega}{2}\right) \int [H'(\bar{k}')]^{1/2} [H''(-\bar{k}')]^{1/2} \times \frac{\delta(|\bar{k}'| - |\bar{k}|)}{|\bar{k}'|} d\bar{k}' \\
 &= \rho_0 \cos^2 \theta \left(\frac{\omega}{2}\right)^2 I\left(\frac{\omega}{2}\right) \int [h(\phi)]^{1/2} [h(180^\circ + \phi)]^{1/2} d\phi \\
 &= \rho_0 \cos^2 \theta \left(\frac{\omega}{2}\right)^2 I\left(\frac{\omega}{2}\right) 0.8 \int_0^{\pi/2} \sin^3 \phi d\phi \\
 &= \frac{0.4}{3} \rho_0 \cos^2 \theta (\omega)^2 I\left(\frac{\omega}{2}\right), \tag{43}
 \end{aligned}$$

where the relations

$$(K_3)^2 (d\phi)^2 = (\omega)^2 I(\omega) H(K), \quad \cos \theta = \frac{|k_3|}{|k|}$$

have been applied. The wavenumber and frequency power spectrum at the interface is

$$S_{pp}(\omega, \bar{k}) = \frac{0.4}{3} \rho_0^2 \cos^4 \theta I^2(\omega) \omega^4, \tag{44}$$

with  $\cos \theta = \frac{|k_3|}{|k|}$ .

Equations (43) and (44) can serve for the boundary conditions ascribed to the surface-wave noise-generating mechanism when computing the noise field for region III.

### Wave Turbulence

Wind generates waves on the surface of the ocean. It may also induce flow motion at the surface and cause vorticity. Therefore, there must be some interaction between the wave motion and the turbulent motion at the surface of the ocean. Based on the energy balance between amplification of vorticity by surface flow motion and dissipation of vorticity by viscosity, it appears that wind-induced turbulence is small (of the third order).<sup>28</sup> However, the acoustic wave is a



second-order quantity, so the wind-induced turbulence may make some contribution in the ambient sea noise level, particularly if it interacts with the surface wave motion that is a first-order quantity.

To evaluate the acoustic pressure due to the interaction of wind-induced turbulence and surface waves, equation (9) is rewritten by including the relevant terms:

$$\begin{aligned}\frac{\partial^2 p}{\partial x_i^2} &\approx -\rho_0 \frac{\partial^2}{\partial x_i \partial x_j} [U_i u_j] + \dots \\ &= -\rho_0 \frac{\partial^2}{\partial x_i \partial x_j} \left[ \frac{\partial \phi}{\partial x_i} u_j \right] + \dots\end{aligned}\quad (45)$$

Applying the same technique used for surface waves by the Fourier-Stieltjes integral representation, we get

$$\begin{aligned}p(\bar{x}, t) &= \int dP(\bar{k}, \omega) e^{i(\omega t - \bar{k} \cdot \bar{r}) - ik_3 x_3} d\bar{k} d\omega, \\ \phi(\bar{x}, t) &= \int d\phi(\bar{K}, \omega) e^{i(\omega t - \bar{K} \cdot \bar{r}) + K_3 x_3} d\bar{K} d\omega, \\ u_j(x, t) &= \int d\bar{U}_j(\bar{K}, \omega) e^{i(\omega t - \bar{K} \cdot \bar{r}) - iK_3 x_3} d\bar{K} d\omega.\end{aligned}\quad (46)$$

In  $\bar{k}$ ,  $\omega$  space

$$\begin{aligned}|k|^2 dP(\omega, \bar{k}) &= -\rho_0 \int [K_i'^2 K_j' + K_i' K_j' (k_i - K_i') \\ &\quad + K_i'^2 (k_j - K_j') + K_i' (k_i - K_i') (k_j - K_j')] d\phi'(\bar{K}', \omega') \\ &\quad d\bar{\omega}_j'(\bar{k} - \bar{K}', \omega - \omega') d\bar{K}' d\omega' \\ &= -\rho_0 \int K_i' k_i k_j d\phi(\bar{K}', \omega') d\bar{U}_j(\bar{k} - \bar{K}', \omega - \omega') d\bar{K}' d\omega'.\end{aligned}\quad (47)$$

The dispersive relation of equation (27) does not apply to wind-induced turbulence. However, simplification of equation (47) is accomplished by considering that there is no correlation between surface waves and turbulence

$$S_{pp}(\omega, \bar{k}) \approx \rho_o^2 \left( \frac{k_3}{|k|} \right)^4 \int K'^2 E_\omega(\omega', \bar{k}') E_T(\omega - \omega', \bar{k} - \bar{k}'), d\omega', d\bar{k}', \quad (48)$$

where

$$E_\omega(\omega', \bar{k}') = I(\omega') H(K') \frac{\delta(|K'| - K_o)}{|K|}$$

and

$$E_T(\omega - \omega', \bar{k} - \bar{k}') = \frac{d\bar{U}_i(\omega - \omega', \bar{k} - \bar{k}') d\bar{U}_j(\omega - \omega', \bar{k} - \bar{k}')}{|K|}$$

the spectrum of surface turbulence, according to the measurement, generally follows Kolmogoroff's Law.<sup>35-37</sup> From the field data,<sup>37,38</sup> the spectrum of turbulence can be estimated to have the form

$$E_T(\omega, \bar{k}) = \frac{.8}{[1 + 2.10^4 |k|^{5/3}][1 + \omega^2]} \quad (49)$$

Using the results from equations (23), (33), and (49), we can employ equation (48) to compute the noise power spectrum due to wave turbulence by numerical integration. The final result is

$$S_{pp}(\omega, \bar{k}) \approx 2.10^{-2} \rho^2 \cos^4 \theta \omega^{-2} U \cos \theta = \frac{|k_3|}{|k|} \quad (50)$$

#### NOISE FIELD COMPUTATION

Disturbances at the air-water interface cause pressure fluctuations at the ocean-surface boundary. The power spectra of such pressure fluctuation attributed to three different types of noise-generating mechanisms have been derived earlier. The noise field in the water column, region III in figure 1, can be computed according to the wave equation,

$$\frac{\partial^2 p}{\partial x_i^2} - \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = 0, \quad (51)$$

where it has been assumed that the depth of surface disturbance  $d$  is much smaller than acoustic wavelength  $\lambda$ . Therefore region III is source free. The surface disturbance can be considered to be a boundary condition imposed at  $x_3 = 0$ .

For a single harmonic disturbance of radian frequency,  $\omega$ , the solution for equation (51) in region III for a water column is

$$p_m = a_m(\bar{k}_m) e^{i\bar{k}_m \cdot \bar{r}} (e^{-ik_{3m} x_3} + b_m e^{ik_{3m} x_3}) \quad (52)$$

and, for a bottom layer, is

$$p_{bm} = c_m(\bar{k}_{bm}) e^{i\bar{k}_{bm} \cdot \bar{r} - ik_{b3m} x_3}, \quad (53)$$

where  $e^{i\omega t}$  in both equations is the suppressed,  $\bar{r} = (x_1, x_2)$ ,  $k_m$  wave number of mode  $m$ . With the boundary condition at  $x_3 = 0$ ,

$$p_{sm}(\bar{k}_{sm}) e^{i\bar{k}_{sm} \cdot \bar{r}} = a_m(\bar{k}_m) e^{i\bar{k}_m \cdot \bar{r}} (1+b_m) \quad (54)$$

and it results in

$$\bar{k}_{srm} = \bar{k}_{rm}$$

and

$$p_{sm}(\bar{k}_{sm}) = a_m(\bar{k}_m) (1+b_m)$$

at  $x_3 = D$

$$\begin{aligned} a_m(\bar{k}_m) e^{i\bar{k}_m \cdot \bar{r}} (e^{-ik_{3m} D} + b_m e^{ik_{3m} D}) \\ = c_m(\bar{k}_{bm}) e^{i\bar{k}_{bm} \cdot \bar{r} - ik_{b3m} D} \end{aligned}$$

and

$$\begin{aligned} a_m(k_m) e^{i\bar{k}_m \cdot \bar{r}} (-e^{-ik_{3m} D} + b_m e^{ik_{3m} D}) \\ = -c_m(\bar{k}_{bm}) e^{i\bar{k}_{bm} \cdot \bar{r} - ik_{b3m} D} \cdot R, \end{aligned}$$

where

$$R = \frac{\rho c}{\rho_b c_b}.$$

In simplified form

$$\bar{k}_m = \bar{k}_{bm}$$

$$a_m \left( e^{-ik_{3m}D} + b_m e^{ik_{3m}D} \right) = c_m e^{-ik_{b3m}D} \quad (55)$$

and

$$a_m \left( -e^{-ik_{3m}D} + b_m e^{ik_{3m}D} \right) = -c_m e^{-ik_{b3m}D}$$

and equations (54) and (55) can be used to determine  $a_m$ ,  $b_m$ , and  $c_m$ . Hence, the pressure field in the water column at a particular depth,  $s$ , for a particular wavenumber,  $k_m$ , can be expressed as

$$p_m(\bar{k}_m) = \frac{p_{sm} (1+R) e^{-ik_{3m}(S-D)} + (1-R) e^{ik_{3m}(S-D)}}{2[\cos k_{3m}D + iR \sin k_{3m}D]} ; \quad (56)$$

the corresponding intensity

$$I_m = p_m p_m^*$$

$$= \frac{p_{sm} p_{sm}^*}{4} \frac{(1+R)^2 + (1-R)^2 + 2(1-R^2) \cos 2k_{3m}(S-D)}{\cos^2 k_{3m}D + R^2 \sin^2 k_{3m}D}$$

$$= \frac{A_{ms}(\bar{k}_m)}{2} \frac{(1+R^2) + (1-R^2) \cos 2k_{3m}(S-D)}{\cos^2 k_{3m}D + R^2 \sin^2 k_{3m}D} , \quad (57)$$

where the power spectrum,  $A_{ms}$ , due to the noise-generating mechanisms caused by surface disturbance, has already been derived from the last section.

In general,  $R < 1$  and  $R^2 \ll 1$ . Equation (57) can be further simplified to

$$I_m \approx \frac{A_{ms}}{2} \frac{1 + \cos 2k_{3m}(S-D)}{\cos^2 k_{3m}D + R^2 \sin^2 k_{3m}D} . \quad (58)$$

The total field intensity at frequency  $\omega$  attributed to all the wave-number contribution will be

$$I = \int I_m d k_m . \quad (59)$$

Here, since the noise fields of different wave numbers arriving at sensor location  $s$  originate from different locations at the surface, incoherence is assumed for the integration expression from equation (59).

The dominant contributing wave number modes in equation (49) are those that meet the conditions

$$\begin{aligned} \cos k_{3m} D &= 0 \\ \text{or} \quad k_{3m} &= (2n+1) \frac{\pi}{2} \frac{1}{D} . \end{aligned} \quad (60)$$

Those modes also have to be the unattenuated modes generated at the surface. They must satisfy

$$\begin{aligned} k_{rm}^2 &= \left( \frac{2\pi}{\lambda} \right)^2 - k_{3m}^2 \\ &= \left( \frac{2\pi}{\lambda} \right)^2 \left[ 1 - \frac{(2n+1)^2}{4} \frac{\lambda^2}{D^2} \right] \geq 0, \end{aligned} \quad (61)$$

$$\text{or} \quad n \leq \frac{2D}{\lambda} - \frac{1}{2} .$$

Hence, the intensity of the noise field at the sensor's depth,  $s$ , in the water column can be approximated by the relation

$$I = \sum_n \frac{A(k_{1n})^2}{2} \frac{1 + \cos \left[ (2n+1) \frac{\pi}{2} \left( 1 - \frac{s}{D} \right) \right]}{R^2} , \quad (62)$$

where  $n$  must meet the conditions given by equations (60) and (61).

The corresponding noise power spectrum measured by the sensors at depth,  $D$ , will be

$$I(\omega) = \phi(\omega) \sum_n \frac{A(\bar{k}_{1n})^2}{2} \frac{1 + \cos \left[ (2n+1) \frac{\pi}{2} \left( 1 - \frac{s}{D} \right) \right]}{R^2} , \quad (63)$$

where  $A(\bar{k}) \phi(\omega)$  is the noise power spectrum generated at the air-water interface and is derived in the last section.

#### MEASUREMENT DATA COMPARISON

Figure 2 was prepared from the processed low frequency ambient noise data collected in the Bermuda area during January 1966 and illustrates the data set reflecting the least interference from a nonwind-dependent noise source.<sup>13</sup> The plotted noise spectrum level was obtained from a bottom-mounted hydrophone, 900 m (2600 ft) below the surface. The noise data are grouped by corresponding wind speed and show distinct profiles above 1 Hz. For the spectrum level below 1 Hz, the data becomes questionable because of system noise interference and deterioration of system response.

The theoretical values of ambient noise attributed to three hypothesized mechanisms are computed according to equation (43) with appropriate parameters (bottom loss data are about 8 dB), and the results are plotted in figure 3 for comparison with the measured data at 20 to 40 knot wind speeds.<sup>39</sup> Although none of the theoretical values match exactly with the measured data, they appear to be of the same order of magnitude. The spectrum shape of wind-induced flow turbulence noise particularly seems to follow that of the measured data. This may indicate that some of the constants assumed in the derivation of the generating mechanism are not quite correct, but the general relationship in the analysis still seems to hold. It appears, judging from the shape of each individual curve with respect to frequency, that the wind turbulence and the surface wave are the dominant cause for wind-induced ambient noise below 7 Hz. Wave turbulence becomes an important noise-generating mechanism for frequencies above 7 Hz.

To further identify the important parameters of the noise-generating mechanisms, figure 4 is a replot of all the measured data and theoretical values with wind speed as the coordinate. The noise level change caused by the surface wave is small, as most energy in wind is converted into low-frequency surface waves when the wind speed increases (Pierson-Moskowitz Model<sup>30</sup>). On the other hand, the noise level change caused by airflow turbulence and turbulence wave interaction covers a wider range and follows almost the same pattern as the measured data. At 5 Hz, these curves indicate that wind turbulence and surface wave noise generation mechanisms provide a reasonably

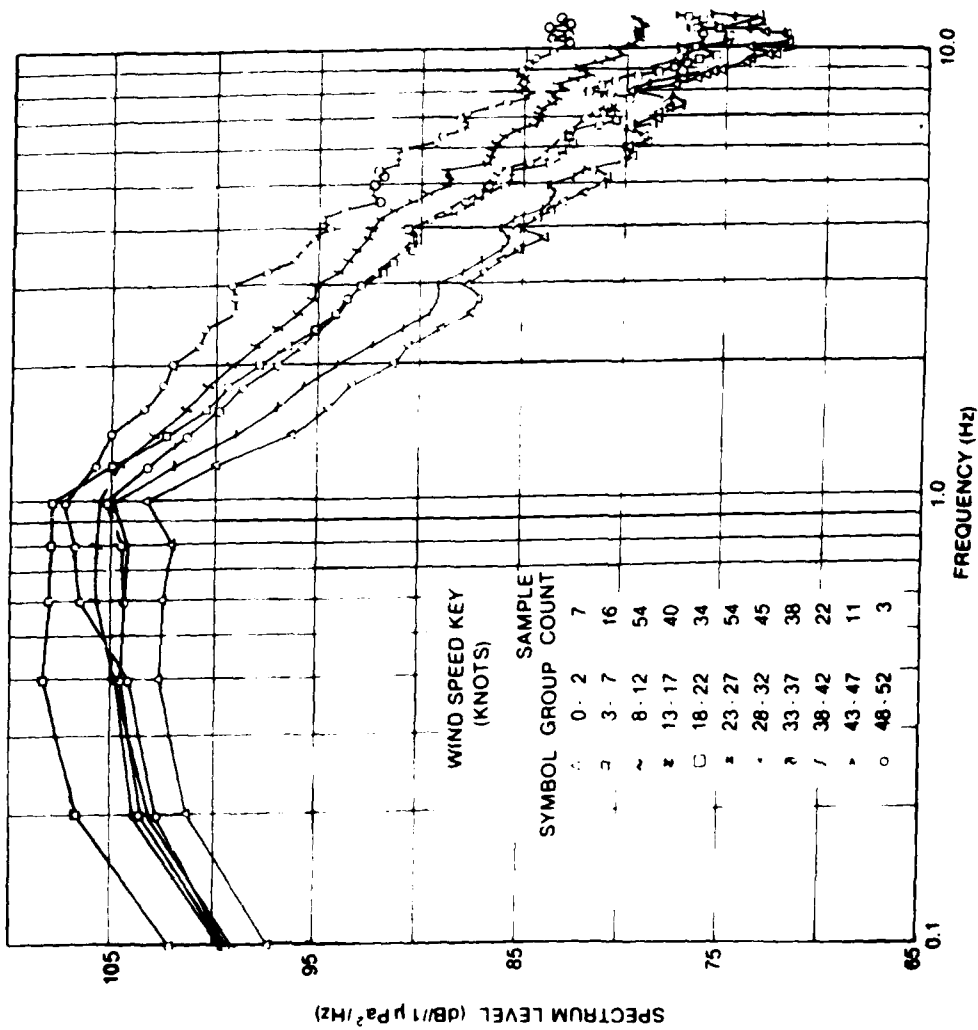


Figure 2. Ambient Noise Spectrum Versus Windspeed Groups  
(Bermuda, 1966, Water Depth 900 m (2600 ft))

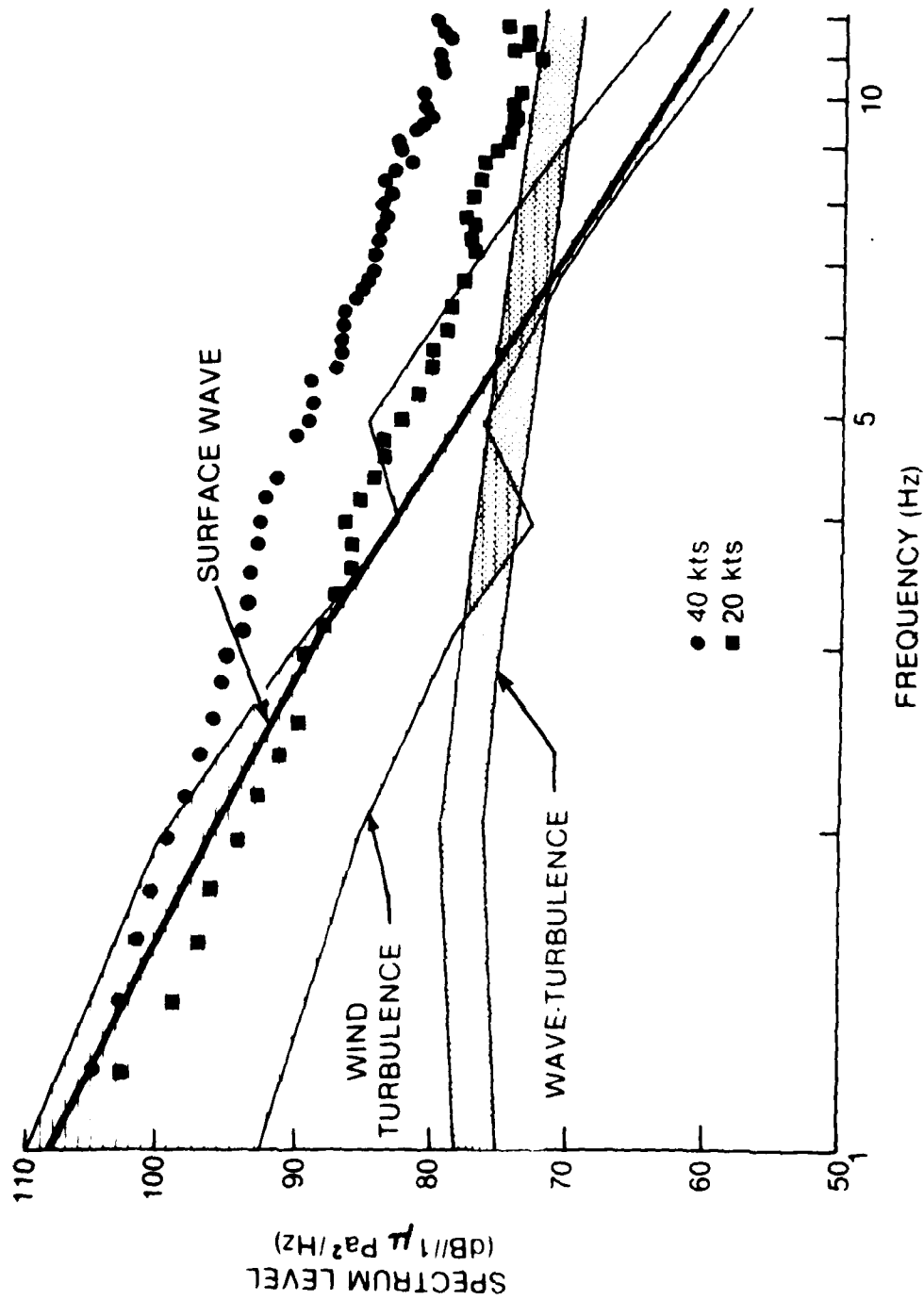


Figure 3. Measurement and Theory (Bermuda, 1966 Water Depth 900 m (2600 ft))



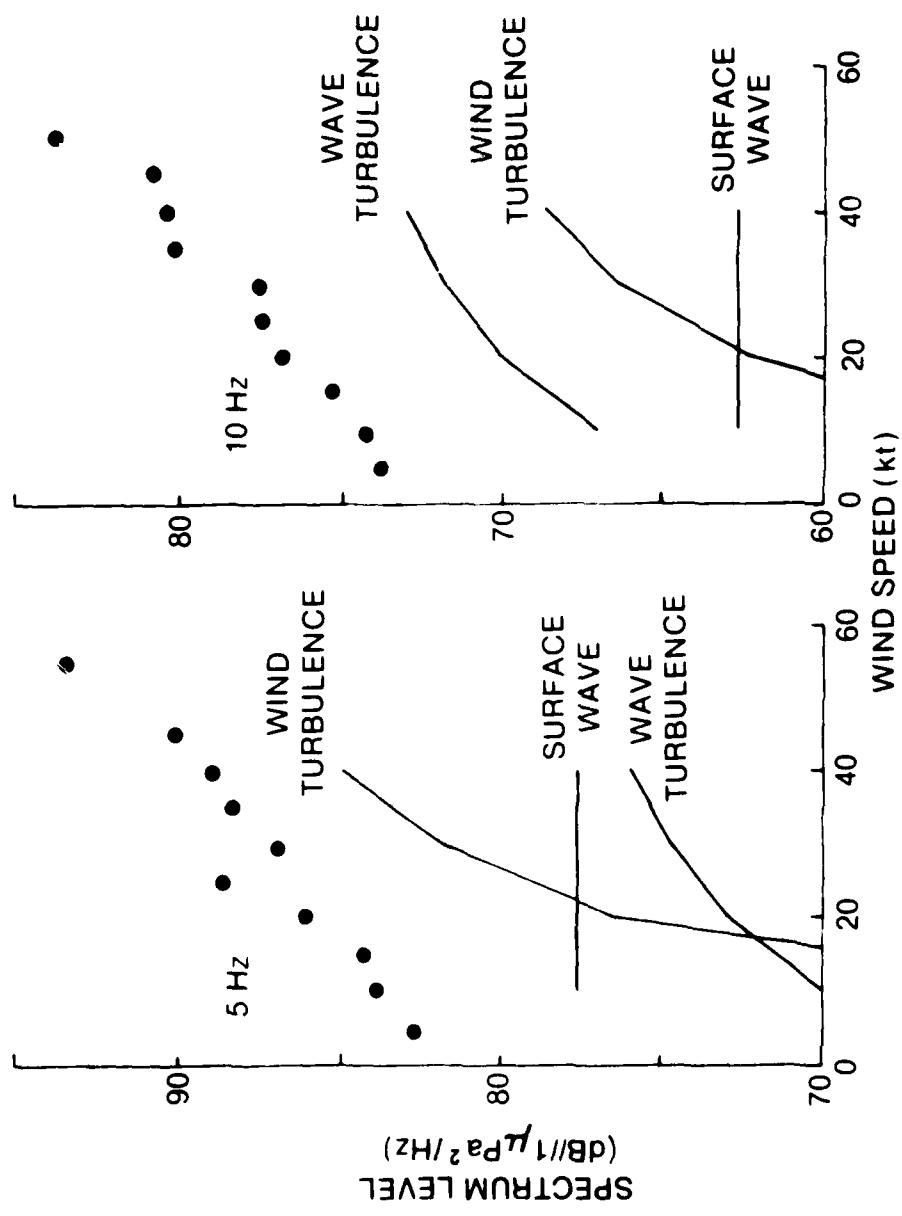


Figure 4. Spectrum Level Versus Windspeed (Bermuda, 1966, Water Depth 900 m (2600 ft))

corrected value of ambient noise level. At 10 Hz, the wave turbulence noise-generation mechanism provides very good predictions on the noise level in comparisons with the measured data.

Comparisons between theory and measurement are also made for data collected at Grand Bank, as shown in figure 5.<sup>3</sup> Agreement of the order of magnitude still appears to hold, but the theoretical values, in general, are lower than the measurement results. In this particular area, it seems that the surface wave mechanism is the dominant noise source.

#### MODELING

Although the theoretical value based on the analysis is not completely in agreement with the measured data, the order of magnitude and spectrum shape provide reasonable support for the hypothesis that the three different noise-generating mechanisms are responsible for wind-induced ambient noise. The main discrepancy is caused by lack of fundamental information on air turbulence, wave directionality, and surface wave turbulence. Experimental data for these parameters in the ocean environment are sparse. However, a semi-empirical model for noise can be constructed by matching the spectrum level with measured data in terms of the parameters identified through theoretical analysis.

There are other variables in formulating a noise model. First, according to equation (42), bottom loss is a very important factor in the final value of the noise field. For a more accurate computation, a computer model of propagation should be incorporated in the noise-field calculation. Other factors affecting the noise level are the geographic location of the site and the time history of wind conditions. For example, if the measurement site is close to the coast or a storm center, it is highly probable that waves propagated in the opposite direction can exist. In this case, the dominant mechanism may be caused by the surface waves. Another deviation from the analysis would be a short duration of a certain wind speed, or a relatively fast wind speed change. The fetch will then be small and the stationary nature of the surface disturbance cannot be assumed. Probably, the predicted noise spectrum level will be overestimated. The shift in wind direction will cause a change in the wave-directional spectrum and, hence, the noise spectrum level.

Formulation of a noise model for computer operation can be based only on the mean of a stochastic process of, in a sense, a determinant model. Since the means are well established for computing the sound field from a given surface source distribution, the desired noise model is the noise-source distribution at the surface (region II in figure 1).

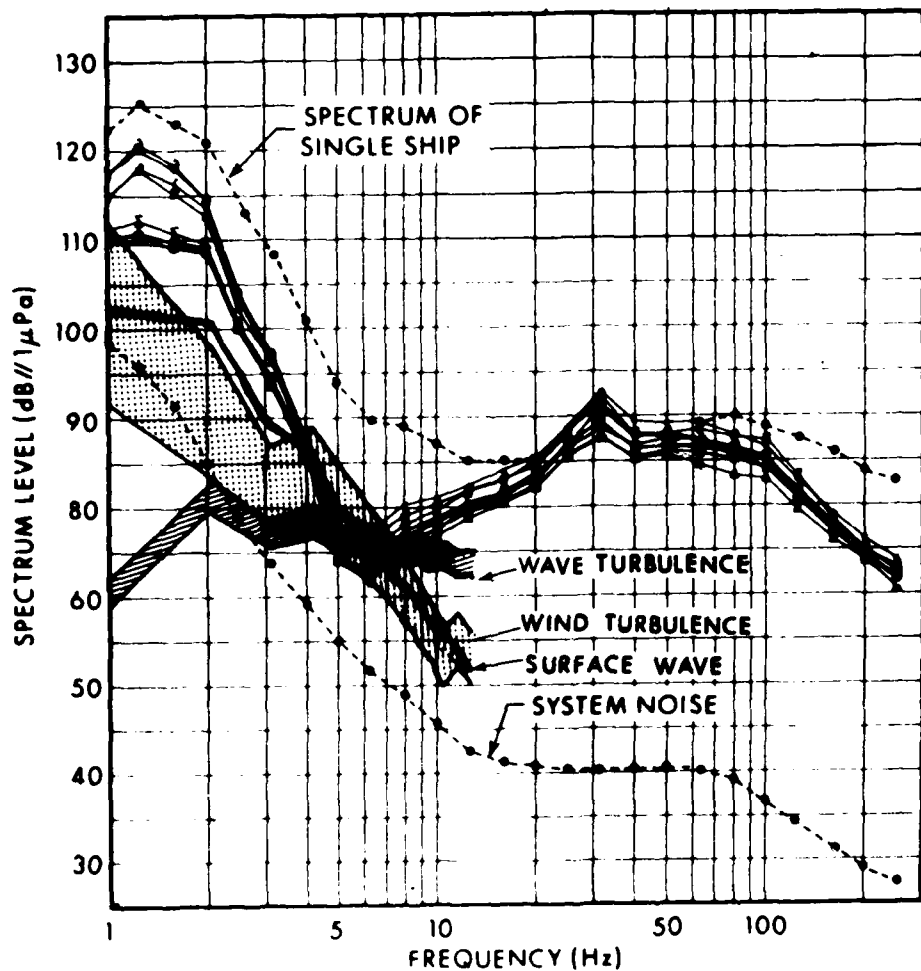


Figure 5. Measurement and Theory (Grand Banks, 1972, Water Depth 1200 m (3684 ft))

From the above consideration and the results of the noise-mechanism analysis, the suggested form of surface-noise-distribution model can be

$$p^2 = \frac{fe^{-0.74g^4/U^4\omega^4}}{(1+\alpha\omega)^6} + \frac{gU^4}{(1+\beta\omega^4)} + \frac{hU}{(1+\gamma\omega^2)} F(\theta), \quad (64)$$

where

$F(\theta)$  = noise radiation pattern ( $\cos^4 \theta$ ), and

$f, g, h, \alpha, \beta, \gamma$  = constants to be determined empirically that are functions of wind duration, direction, and site topography.

The first term is attributed to the surface-wave-generating mechanism. The second and the third are caused by wind turbulence and wave turbulence, respectively. More data are needed for systematic analysis in order to validate these controlling constants of the noise model.

#### CONCLUSIONS

Three noise-generating mechanisms have been examined at the air-water interface of the ocean surface. With the propagation condition and boundary effect incorporated, the derived noise field based on those three mechanisms provide a reasonable explanation of the measured noise data for various wind conditions. The physical argument used in the derivation of wind-induced low-frequency noise enables simplification of the mathematical operation and provides a better understanding of the observed phenomenon. A noise model can, therefore, be formulated from the derived relationship with appropriate adjustment of parameters.

There are some basic problems required for further study. The exact turbulent structure above and below the surface of the ocean is not fully explored for various wind conditions. Such knowledge is needed to extend the current analysis to nonstationary conditions of ocean surface; i.e., the transition stage of a passing or distant storm.

The results from the current analyses indicate that the noise spectrum level is sensitive to bottom properties. This means wind-induced noise might be geographically dependent. There are no available data measured at other locations to verify such conclusions. Some measured low frequency data have been published but there is a lack of supporting wind speed time series information.<sup>40, 41</sup> The Grand Banks

data show a different slope and higher noise level when compared with the Bermuda data.<sup>3</sup> It appears that the surface-wave-generating mechanism is the dominant source for the Grand Banks data. Since the bottom loss has never been measured at either site for the 1 to 10 Hz frequency range, no definite conclusion can be reached on its effect on the measured ambient-noise level.

It has also been hypothesized that wind-induced deep sea currents might be another noise-generating source. However, the observation in the Bermuda area indicates that the magnitude is very small (less than 1 knot) with no noticeable wind speed correlation.<sup>42</sup> The hydrophones used for noise measurements in the Bermuda area were laid or placed on the bottom. Therefore, there should not be any strumming problem with the cable, and any turbulence around the hydrophone would probably be small with such a small current.

Some other noise-generating mechanisms (such as microseismics, seaquakes, shipping, and biological effects) may also be responsible for low-frequency noise; however, they are not correlated with wind condition. The analysis discussed here does not include such a study.

The result of the current study may be extended to other frequency ranges of the noise spectrum, but this would require further examination. Because both airflow turbulence and surface-wave generated noise have a high order of negative slope, their contribution to noise at a high frequency may be small and surface wave turbulence may be assumed to be the dominant mechanism. But, in general, from 10 to 200 Hz, other non-wind-dependent noise sources mask the whole spectrum. Wind-dependent noise becomes appreciable only above 200 Hz. It has been postulated that bubbles and water drops are the causes.<sup>43,44,45</sup> Further study certainly is needed to explore such a possibility.

The theoretical analysis discussed in this report does not result in a precise picture of what occurs in a real ocean; however, it has identified some of the more important parameters in the wind-induced noise-generating mechanism and it also serves as a guideline for future noise measurement. If such information as wind direction, surface turbulence, wave direction, and wave turbulence can be collected simultaneously when making noise measurements the ocean will be better understood.

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